**Dimension Reduction- Recall Maths**

**The concept of mean, median and mode**

* The **mean**is the [average](https://www.statisticshowto.com/arithmetic-mean/)of a data set.
* The **mode**is the most common number in a data set.
* The **median**is the middle of the set of numbers.
* Mean of the sampling distribution: the center of a [probability distribution](https://www.statisticshowto.com/probability-and-statistics/statistics-definitions/probability-distribution/), especially with respect to the [Central Limit Theorem](https://www.statisticshowto.com/probability-and-statistics/normal-distributions/central-limit-theorem-definition-examples/). It’s an average (of sorts) of a set of distributions.
* [Sample mean](https://www.statisticshowto.com/probability-and-statistics/statistics-definitions/sample-mean/): the average value in a [sample](https://www.statisticshowto.com/sample/).
* [Population mean](https://www.statisticshowto.com/population-mean/): the average value in a [population](https://www.statisticshowto.com/what-is-a-population/).

Practice1)Find the mean, median and mode of the give data

1. 1,2,3,4,4,5,5,6,6,7,8,9,9,9,10

**What is Covariance?**

In mathematics and [statistics](https://corporatefinanceinstitute.com/resources/knowledge/basic-statistics-concepts/), covariance is a measure of the relationship between two random variables. The metric evaluates how much – to what extent – the variables change together. In other words, it is essentially a measure of the variance between two variables. However, the metric does not assess the dependency between variables.

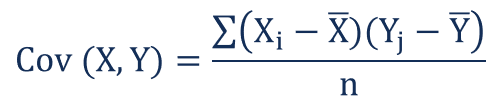
Unlike the correlation coefficient, covariance is measured in units. The units are computed by multiplying the units of the two variables. The variance can take any positive or negative values. The values are interpreted as follows:

* **Positive covariance**: Indicates that two variables tend to move in the same direction.
* **Negative covariance**: Reveals that two variables tend to move in inverse directions.

In [finance](https://corporatefinanceinstitute.com/resources/knowledge/finance/), the concept is primarily used in portfolio theory. One of its most common applications in portfolio theory is the [diversification](https://corporatefinanceinstitute.com/resources/knowledge/strategy/diversification/) method, using the covariance between assets in a portfolio. By choosing assets that do not exhibit a high positive covariance with each other, the unsystematic risk can be partially eliminated.

### **Formula for Covariance**

The covariance formula is similar to the formula for correlation and deals with the calculation of data points from the average value in a dataset. For example, the covariance between two random variables X and Y can be calculated using the following formula (for population):



Where:

* **Xi**– the values of the X-variable
* **Yj**– the values of the Y-variable
* **X̄**– the mean (average) of the X-variable
* **Ȳ** – the mean (average) of the Y-variable
* **n** – the number of data points

## What are Eigenvalues?

The eigenvalue is explained to be a scalar associated with a linear set of equations which, when multiplied by a nonzero vector, equals to the vector obtained by transformation operating on the vector.

Let us consider k x k square matrix A and v be a vector, then λ is a scalar quantity represented in the following way:

AV = λV

Here, λ is considered to be the eigenvalue of matrix A.

The above equation can also be written as:

(A – λI) = 0

Where “I” is the identity matrix of the same order as A.

This equation can be represented in the determinant of matrix form.

|A–λI|=0

The above relation enables us to calculate eigenvalues λ easily.

## Steps to Find Eigenvalues of a Matrix

In order to find the eigenvalues of a [matrix](https://byjus.com/jee/matrices/), follow the steps below:

**Step 1:** Make sure the given matrix A is a square matrix. Also, determine the identity matrix I of the same order.

**Step 2:** Estimate the matrix A – λI, where λ is a scalar quantity.

**Step 3:**Find the determinant of matrix A – λI and equate it to zero.

**Step 4:** From the equation thus obtained, calculate all the possible values of λ, which are the required eigenvalues of matrix A.

## Decomposition of Eigenvalues

The computation of eigenvalues and [eigenvectors](https://byjus.com/jee/eigenvectors-of-a-matrix/) for a square matrix is known as eigenvalue decomposition. When we process a square matrix and estimate its eigenvalue equation, and using the estimation of eigenvalues is done, this process is formally termed as eigenvalue decomposition of the matrix.

Eigenvalues so obtained are usually denoted by λ1, λ2, ….. or e1, e2, ….

## Properties on Eigenvalues

Let A be a matrix with eigenvalues λ1, λ2, …, λn.

The following are the properties of eigenvalues.

1. The trace of A, defined as the sum of its diagonal elements, is also the sum of all eigenvalues,

tr(A)=∑i=1naii=∑i=1nλi=λ1+λ2+⋯+λn.

2. The determinant of A is the product of all its eigenvalues,

det(A)=∏i=1nλi=λ1λ2⋯λn.

3. The eigenvalues of the kth power of A; that is, the eigenvalues of Ak, for any positive integer k, are

λ1k,…,λnk.

4. Matrix A is invertible if and only if every eigenvalue is nonzero.

5. If A is invertible, then the eigenvalues of A-1 are

1λ1,…,1λn

and each eigenvalue’s geometric multiplicity coincides. The characteristic polynomial of the inverse is the reciprocal polynomial of the original, and the eigenvalues share the same algebraic multiplicity.

6. If A is equal to its conjugate transpose, or equivalently if A is Hermitian, then every eigenvalue is real. The same is true of any symmetric real matrix.

7. If A is not only Hermitian but also a positive-definite, positive-semidefinite, negative-definite, or negative-semidefinite, then every eigenvalue is positive or non-negative, negative, or non-positive, respectively.

8. If A is unitary, every eigenvalue has absolute value |λi| = 1.

9. If A is an n × n matrix and {λ1, λ2, …, λk} are its eigenvalues, then the eigenvalues of the matrix I + A (where I is the identity matrix) are {λ1 + 1, λ2 + 1, …, λk + 1}.

## Solved Problems on Eigenvalues

Sample problems based on eigenvalue are given below:

**Example 1**: Find the eigenvalues for the following matrix.

A=[2 1

4 5]

**Solution:**

Given,

A=[2 1

4 5]

A−λI=[2−λ 1

4 5−λ]

|A – λI| = 0

⇒|2−λ 1

4 5−λ|=0

(2-λ)(5-λ) – 4 = 0

 ⇒ 10- 5λ – 2λ +λ2-4 = 0

⇒ λ2-7λ +6 = 0

⇒( λ-6)(λ-1) = 0

⇒λ = 6 or λ= 1

Hence the required eigenvalues are 6 and 1.

**Example 2:** Find the eigenvalues of the matrix

A=[2 0

−1 1].

**Solution –**

Given,

A=[2 0

−1 1]

Then,

A–λI=[2–λ 0

−1 1−λ]

|A – λI| = 0

(2 – λ) (1 – λ) – 0 = 0

(2 – λ)(1 – λ) = 0

λ = 1, 2

These are required eigenvalues.

**Example 3:** Calculate the eigenvalue equation and eigenvalues for the following matrix –

[1 0 0

0 −1 2

2 0 0]

**Solution –**

Let

A=[1000−12200]

A–λI=[1−λ000−1−λ2200–λ]

We can calculate eigenvalues from the following equation:

|A – λI| = 0

(1 – λ) [(- 1 – λ)(- λ) – 0] – 0 + 0 = 0

λ (1 – λ) (1 + λ) = 0

From this equation, we can estimate eigenvalues which are –

λ = 0, 1, -1.

Let **A** be any square matrix. A non-zero vector **v** is an **eigenvector** of **A** if  
  
**Av =** λ**v**  
  
for some number λ, called the corresponding **eigenvalue**.

## How to find the eigenvalues and eigenvectors of a 2x2 matrix

1. Set up the **characteristic equation**, using |**A** − λ**I**| = 0
2. **Solve** the characteristic equation, giving us the **eigenvalues** (2 eigenvalues for a 2x2 system)
3. **Substitute** the eigenvalues into the two equations given by **A** − λ**I**
4. Choose a convenient value for x1, then find x2
5. The resulting values form the corresponding **eigenvectors** of **A**(2 eigenvectors for a 2x2 system)

There is no single **eigenvector formula** as such - it's more of a sset of steps that we need to go through to find the eigenvalues and eigenvectors.

Let's have a look at some examples.

https://www.intmath.com/matrices-determinants/7-eigenvalues-eigenvectors.php

## How many eigenvalues and eigenvectors?

In the above example, we were dealing with a 2×2 system, and we found 2 eigenvalues and 2 corresponding eigenvectors.

If we had a 3×3 system, we would have found 3 eigenvalues and 3 corresponding eigenvectors.

In general, *n*×*n* system will produce *n* eigenvalues and *n* corresponding eigenvectors.